describing heat-transfer processes was developed by A.S. Predvoditelev [6].

In the present communication a geometrical interpretation of heat-transfer processes governed by equation (2) is developed. This trend follows from the works by Poincaré [8], Barankin [9], Robertson [10].

2. Nonuniformly heated medium with the temperature $T(x, y, z, t)$ induces a Reimannian space with a metric of the form

$$ds^2 = \frac{1}{(1 + hT)^2}\,[w^2 dt^2 - (dx^2 + dy^2 + dz^2)]. \qquad (3)$$

The metric described by expression (3) is equivalent to the invariant $ds^2 = w^2dt^2 - dx^2 - dy^2 - dz^2$ of the acoustic theory of relativity [11]. In the metric the sound velocity w is taken as a fundamental velocity. Expression (3) implies that the lengths are measured by a calibration rod with thermal expansion coefficient h.

The Riemannian space with a metric of the form (3) may be described by mean curvature $\bar{K}$. As is known from the

$$\bar{K} = \frac{h}{2w^2}\,\Box^2 T + O(h^2) \qquad (4)$$

where $\Box^2$ is the d'Alembert operator in equation (2).

With the help of equation (2) a final form of the expression for a mean curvature is obtained

$$\bar{K} = \frac{hp}{2\lambda}. \qquad (5)$$

Thus, the heat transfer problem for hyperbolic equation (2) is reduced to finding a mean curvature of the Riemannian space with metric (3). As follows from equation (5), the curvature of this space is proportional to the function of heat sources. This is similar to curving of the space caused by the material mass effect in the Einstein gravitation theory.

To conclude the note, the authors would like to express their gratitude to Prof. A. V. Luikov for his fruitful discussions.

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Int. J. Heat Mass Transfer. Vol. 15, pp. 1284–1287. Pergamon Press 1972. Printed in Great Britain

REYNOLDS FLUX AND DANCKWERTS SURFACE RENEWAL THEORY

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(*Received* 21 *July* 1971)

NOMENCLATURE

A, dA,	area of surface;
C,	concentration;
C_p,	specific heat at constant pressure;
D,	diffusion coefficient;
h,	heat-transfer coefficient;

h_D,	mass transfer coefficient;
K,	thermal conductivity;
t,	time;
U,	velocity of fluid;
R,	entity size parameter;
α,	thermal diffusion coefficient;
ε,	Reynolds flux;
μ,	dynamic viscosity;
ν,	kinematic viscosity;
ρ,	density;
τ,	shear stress;
θ,	temperature difference between fluid and transfer surface.
Subscripts	
∞,	in bulk fluid;
0,	at transfer surface.

A THEORY of the observed correlation between convective heat transfer and convective momentum transfer appears first to have been suggested by Reynolds [1].* Reynolds theory assumed that both such processes were due to the motion of fluid particles in lateral directions relative to the mean axial flow of the fluid. In 1950, Silver [2] suggested that by deducing the amount of lateral motion from known experimental results of heat transfer *or* of frictional resistance, the limit rates of mass transfer access to a surface could be calculated. From 1950 to 1954 this was applied by him to the calculation of combustion rates of fuel particles and in firebeds [3, 4], and later to the calculation of condensation rates and associated pressure drop [5]. Meanwhile similar applications of Reynolds theory had been developed by Spalding [6] who coined the term Reynolds Flux as a name for the assumed lateral motion of fluid particles. Both Silver and Spalding recognised that they were developing a concept initiated by Reynolds.

However, in 1951 Danckwerts [7] in discussing gas absorption in liquid films suggested a process of replacement of liquid surface by lateral flow of fresh liquid, and Mickley Fairbanks [8] adapted this to a discussion of heat transfer in fluidized beds. Danckwerts coined the term "surface renewal mechanism" to describe his idea. There is no doubt, however, that the underlying concept is an assumed lateral flux, yet neither Danckwerts' nor Mickley and Fairbanks appear to have recognized the identity of this concept with that of Reynolds. The explanation of this may have been that, as discussed in the Reynolds Centenary volume [9], Reynolds actual theoretical model was largely neglected because of the loose term "Reynolds Analogy" which had been commonly used to describe his ideas, and it was only from [2] onwards (1950), that interest was directed to the flux concept. Alternatively it may simply illustrate the common difficulty of "information transfer" from one field of study to another.

We have examined several references to surface renewal theory [10–12] including one very recently in this journal and in no case have we found comment on its relation to Reynolds theory.

It seems therefore worthwhile to point this out briefly.

Where Reynolds referred to "non-molecular fluid particles", Danckwerts speaks of "eddies exposing fresh surface" from the interior of the liquid, and Silver adopts the general term "entities" to describe the elements whose lateral motion constitutes the lateral flux.

In Reynolds original theory, the lateral flux was assumed to reach equilibrium with wall surface conditions instantaneously on reaching the wall. This simple assumption was on the whole maintained in the further work by Silver and by Spalding, although in 1966 Silver [13] discussed the possibility that the return flux from the wall might not have reached equilibrium with wall conditions. Danckwerts, however, included the possibility of incomplete equilibration of flux with wall conditions right from the start.

A duration time of the entity while in contact with the surface is postulated, and diffusion equations for energy, mass, or momentum can be applied for these duration times. A statistical mean life and corresponding mean transfer rates can be assessed. This has been the pattern of development of surface renewal theory.

It is seen therefore that essentially surface renewal theory is one form of refinement of Reynolds theory, using the same basic concept of a lateral flux, but permitting adjustments through the concept of extent of equilibration with wall conditions.

We have therefore used the ideas of surface renewal to develop an improved two parameter form of the Reynolds theory. We retain the Reynolds Flux (ε) as one parameter but introduce a size parameter R which physically corresponds to a normal dimension of the "eddies" or "entities" in contact with the transport surface or wall. As in Danckwerts formulation [8] molecular transport occurs to the entities in contact with the solid surface.

Thus the transport processes within fluid elements are defined by:

$$\left.\begin{aligned} &\alpha\frac{\partial^2\theta}{\partial x^2} = \frac{\partial\theta}{\partial t} &&\text{—heat transfer}\\ &\nu\frac{\partial^2 U}{\partial x^2} = \frac{\partial U}{\partial t} &&\text{—momentum transfer}\\ &D\frac{\partial^2 C}{\partial x^2} = \frac{\partial C}{\partial t} &&\text{—mass transfer}\end{aligned}\right\}\quad(1)$$

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* Footnote: This is I think correct, although Thomson (Lord Kelvin) was on the same track earlier. In 1856 commenting on a paper by Joule he referred to "clogging of the water caused by the heating particles not being quickly enough carried away and replaced by cooler ones". —R.S.S.

with boundary conditions

$$t = 0\; x \geqslant 0\; \theta = \theta_\infty\; U = U_\infty\; C = C_\infty$$
$$t > 0\; x = 0\; \theta = 0\quad U = 0\quad C = C_0$$
$$x = \infty\; \theta = \theta_\infty\; U = U_\infty\; C = C_\infty$$

The solutions of these equations give instantaneous transport coefficients at the solid surface.

$$\left.\begin{aligned} h &= \left[\frac{K\rho C_p}{\pi t}\right]^{\frac{1}{2}} && \text{—heat transfer} \\ \tau &= \left[\frac{\mu^2 U^2}{\pi \nu t}\right]^{\frac{1}{2}} && \text{—momentum transfer} \\ h_D &= \frac{D}{[\pi D t]^{\frac{1}{2}}} && \text{—mass transport}. \end{aligned}\right\} \quad (2)$$

To obtain average values of these coefficients it is necessary to obtain some surface age distribution $\phi(t)$. It is at this point that our presentation diverges from that of Danckwerts [8].

The mass of surface $\mathrm{d}A$, having an age between t and $t + \mathrm{d}t$ is simply $\rho R\, \mathrm{d}A\, \phi(t)\, \mathrm{d}t$.

Also the rate of decrease of surface of any age is equal to the rate at which surface of that age is replaced.

Rate of replacement of surface of age t to $t + \mathrm{d}t$ is $\varepsilon.\ \phi(t)\, \mathrm{d}t$ whence

$$\frac{-\mathrm{d}\phi(t)}{\mathrm{d}t} \rho R = \varepsilon \phi(t). \quad (3)$$

This gives the distribution function

$$\phi(t) = \frac{\varepsilon}{\rho R} \exp\left(\frac{-\varepsilon}{\rho R} t\right). \quad (4)$$

Now the heat transfer coefficient for the surface is the average of all the local coefficients

$$h = \left[\frac{K\rho C_p}{\pi}\right]^{\frac{1}{2}} \frac{\varepsilon}{\rho R} \int_0^\infty t^{-\frac{1}{2}} \exp\left(\frac{-\varepsilon}{\rho R} t\right) \mathrm{d}t = \left[\frac{K C_p \varepsilon}{R}\right]^{\frac{1}{2}}. \quad (5a)$$

$$\left.\begin{aligned} &\text{Similarly } \tau = U_\infty \left[\frac{\mu \varepsilon}{R}\right]^{\frac{1}{2}} \\ &\text{and } h_D = \left[\frac{D\varepsilon}{\rho R}\right]^{\frac{1}{2}}. \end{aligned}\right\} \quad (5b)$$

The mean age of entities at the surface is

$$\bar{t} = \int_0^\infty t\, \phi(t)\, \mathrm{d}t = \frac{\rho R}{\varepsilon}. \quad (6)$$

The parameter R in this model represents a surface layer depth and since the diffusion equations have been solved for semi-infinite boundary conditions it would seem that R must be greater than the depth to which the molecular diffusion process has penetrated, i.e.

$$\left.\begin{aligned} R &> (\alpha \bar{t})^{\frac{1}{2}} \\ R &> (\nu \bar{t})^{\frac{1}{2}} \\ R &> (D \bar{t})^{\frac{1}{2}} \end{aligned}\right\} \quad (7)$$

Substituting for $\bar{t}$ from (6)

$$\left.\begin{aligned} &R > \frac{K}{C_p \varepsilon} \\ &\text{or } R > \frac{\mu}{\varepsilon} \\ &\text{or } R > \frac{D_p}{\varepsilon}. \end{aligned}\right\} \quad (8)$$

If these are substituted into the corresponding transport coefficients we obtain

$$\left.\begin{aligned} h &= \left[\frac{K C_p \varepsilon}{R}\right]^{\frac{1}{2}} < \varepsilon C_p \\ \tau &= U_\infty \left[\frac{\mu \varepsilon}{R}\right]^{\frac{1}{2}} < \varepsilon U \\ h_D &= \left[\frac{D\varepsilon}{\rho R}\right]^{\frac{1}{2}} < \frac{\varepsilon}{\rho}. \end{aligned}\right\} \quad (9)$$

Equations (9) merely indicate that the normal Reynolds Theory transport coefficients represent the maximum possible values obtainable.

Physically R might be taken as the depth of the laminar sublayer in turbulent flow. However, the theory of turbulence by Tyldesley and Silver [14] utilizes the entity concept with an entity size related to the microscale of the turbulence as one of the parameters describing the turbulence, and it may later be possible to introduce R in that context.

Experimentally the data from anemometry experiments by Laufer [15] and Wells, Harkness, and Meyer [16] suggest that in ducts or pipes both the depth of laminar sublayer and the turbulent microscale near the walls of the duct are of same order.

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